Name ____________________ Date ____________

LESSON 12.3

Study Guide

For use with pages 810–817

GOAL **Find surface areas of pyramids and cones.**

Vocabulary

A **pyramid** is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex.

The **vertex of the pyramid** is the common vertex of the lateral faces of the pyramid.

A **regular pyramid** has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base.

The **slant height** of a regular pyramid is the height of a lateral face of the regular pyramid.

Theorem 12.4 Surface Area of a Regular Pyramid: The surface area S of a regular pyramid is $S = B + \frac{1}{2}P\ell$, where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.

A **cone** has a circular base and a **vertex** that is not in the same plane as the base.

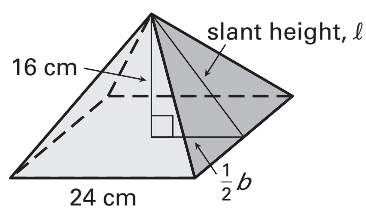
A **right cone** is a cone where the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge.

Theorem 12.5 Surface Area of a Right Cone: The surface area S of a right cone is $S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$, where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.

EXAMPLE 1 Find the area of a lateral face of a pyramid

A regular square pyramid has a height of 16 centimeters and a base edge length of 24 centimeters. Find the area of each lateral face of the pyramid.



Solution

Use the Pythagorean Theorem to find the slant height ℓ.

$\ell^2 = h^2 + \left(\frac{1}{2}b\right)^2$ Write formula.

$\ell^2 = 16^2 + 12^2 = 400$ Substitute for h and $\frac{1}{2}b$ and simplify.

$\ell = 20$ Find the positive square root.

The area of each triangular face is $A = \frac{1}{2}b\ell = (12)(20) = 240 \text{ cm}^2$.

LESSON 12.3

Study Guide *continued*

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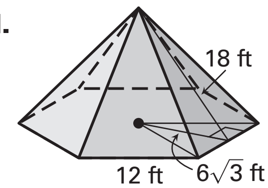
EXAMPLE 2 Find the surface area of a pyramid

Find the surface area of the regular hexagonal pyramid.



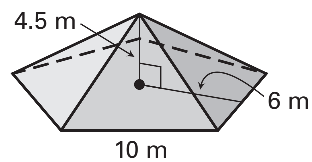
First, find the area of the base using the formula for the area of a regular polygon, $\frac{1}{2}aP$. The apothem a of the hexagon is $6\sqrt{3}$ feet and the perimeter P is $6 \cdot 12 = 72$ feet. So, the area of the base B is $\frac{1}{2}(6\sqrt{3})(72) = 216\sqrt{3}$ square feet. Then, find the surface area.

$S = B + \frac{1}{2}P\ell$ Formula for surface area of a regular pyramid

$S = 216\sqrt{3} + \frac{1}{2}(72)(18) \approx 1022.12$ Substitute known values and simplify.

The surface area of the regular hexagonal pyramid is about 1022.12 square feet.

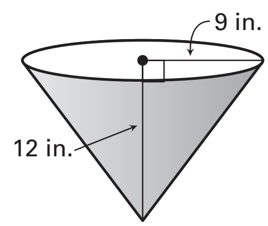
Exercises for Examples 1 and 2



1. Find the area of each lateral face of the regular pentagonal pyramid shown.
2. Find the surface area of the regular pentagonal pyramid shown.

EXAMPLE 3 Find the lateral area and surface area of a cone

Find the lateral area and surface area of the cone.



To find the slant height ℓ, use the Pythagorean Theorem.

$\ell^2 = 12^2 + 9^2 = 225$, so $\ell \approx 15$ inches.

Find the lateral area.

Lateral area $= \pi r\ell = \pi(9)(15) \approx 424.1$

Find the surface area.

$S = \pi r^2 + \pi r\ell = \pi(9)^2 + \pi(9)(15) = 216\pi \approx 678.6$

The lateral area of the right cone is about 424.1 square inches and the surface area is about 678.6 square inches.

Exercises for Example 3

3. Find the lateral area of the right cone shown.
4. Find the surface area of the right cone shown.