Name _____ Date _____

LESSON 8.5

Study Guide

For use with pages 542–549

GOAL **Use properties of trapezoids and kites.**

Vocabulary

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases.** For each of the bases of a trapezoid, there is a pair of **base angles,** which are the two angles that have that base as a side.

The nonparallel sides of a trapezoid are the **legs** of the trapezoid. If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid.** The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs.

A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

Theorem 8.14: If a trapezoid is isosceles, then each pair of base angles is congruent.

Theorem 8.15: If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

Theorem 8.16: A trapezoid is isosceles if and only if its diagonals are congruent.

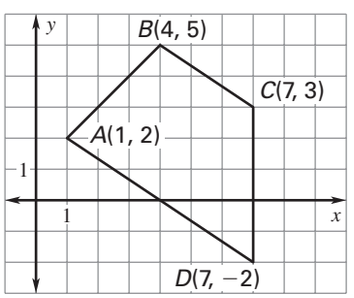
Theorem 8.17 Midsegment Theorem for Trapezoids:
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

Theorem 8.18: If a quadrilateral is a kite, then its diagonals are perpendicular.

Theorem 8.19: If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

EXAMPLE 1 ## Use a coordinate plane

Show that *ABCD* is a trapezoid.



Solution

Slope of $\overline{BC} = \frac{3-5}{7-4} = -\frac{2}{3}$

Slope of $\overline{AD} = \frac{-2-2}{7-1} = -\frac{2}{3}$

Slope of $\overline{AB} = \frac{5-2}{4-1} = 1$

Slope of $\overline{CD} = \frac{-2-3}{7-7} = \frac{-5}{0}$ Undefined

$\overline{BC}$ and $\overline{AD}$ have equal slopes, so they are parallel. $\overline{AB}$ and $\overline{CD}$ are not parallel. Because $ABCD$ has exactly one pair of parallel sides, it is a trapezoid.

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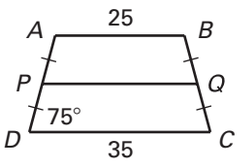
Exercise for Example 1

1. The vertices of $ABCD$ are $A(-5, 6)$, $B(1, 3)$, $C(0, 0)$, and $D(-7, 0)$. Show that $ABCD$ is a trapezoid.

EXAMPLE 2 Use properties of trapezoids

In the diagram, *ABCD* is an isosceles trapezoid and $\overline{PQ}$ is the midsegment.



a. Find $m\angle B$. **b.** Find PQ.

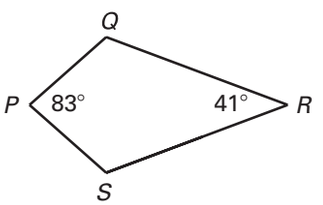
Solution

a. Because $\angle D$ and $\angle A$ are consecutive interior angles formed by $\overleftrightarrow{AD}$ intersecting two parallel lines, they are supplementary. So, $m\angle A = 180° - 75° = 105°$. By Theorem 8.14, $\angle A \cong \angle B$. So, $m\angle B = 105°$.

b. By Theorem 8.17, $PQ = \frac{1}{2}(AB + CD) = \frac{1}{2}(25 + 35) = \frac{1}{2}(60) = 30$.

EXAMPLE 3 Use properties of kites

In the diagram, *PQRS* is a kite. Find $m\angle Q$.



Solution

By Theorem 8.19, $PQRS$ has exactly one pair of congruent opposite angles. Because $\angle P \ncong \angle R$, $\angle Q$ and $\angle S$ must be congruent. So, $m\angle Q = m\angle S$.

$m\angle Q + m\angle S + 83° + 41° = 360°$ Corollary to Theorem 8.1

$m\angle Q + m\angle Q + 83° + 41° = 360°$ Substitute $m\angle Q$ for $m\angle S$.

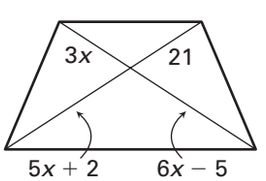
$2(m\angle Q) + 124° = 360°$ Combine like terms.

$m\angle Q = 118°$ Solve for $m\angle Q$.

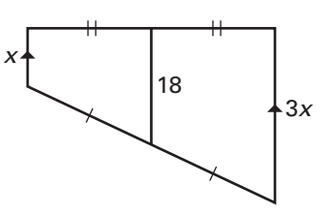
Exercises for Examples 2 and 3

Find the value of *x*.

2.



3.



4. In a kite, the measures of the angles are $6x°$, 24°, 84°, and 126°. Find the value of x. What are the measures of the angles that are congruent?